

PariTwine

A bridge between PARI/GP and the GNU MP universe

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(Software written together with Fredrik Johansson)

MPFR/MPC/iRRAM Workshop, Trier, 21 November 2018



Motivation

- Use C code for evaluating two-dimensional ϑ -functions from inside GP.

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 - ▶ MPFR
 - ▶ MPC
 - ▶ MPFRGX, CM, CMH
 - ▶ FPLLL
 - ▶ ...

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- Create a bridge between the GNU MP universe and PARI/GP.
- Create a FFI for the GNU MP libraries in PARI/GP.

- Software for algebra and number theory
- Written since 1985 in Bordeaux
- Number theory backend of SageMath
- C library (PARI) and REPL/command interpreter (GP)

Mini-demo GP

- 1 Installation
- 2 Conversion of numbers
- 3 Wrapped library functions
- 4 Calling functions from GP

<https://paritwine.multiprecision.org/>

- Version 0.0.1 of 2014 — outdated, do not use.
- Development version
 - ▶ `git clone https://scm.gforge.inria.fr/anonscm/git/pari-gnump/pari-gnump.git`
 - ▶ `autoreconf -vfi`
 - ▶ `./configure`
 - ▶ `make`
 - ▶ `make check`
 - ▶ `make install`
 - ▶ `make pdf; xpdf doc/paritwine.pdf`

- 1 Installation
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- PARI: `t_INT`, `t_FRAC`, `t_REAL`, `t_COMPLEX`

- ▶ stores numbers on the PARI stack
- ▶ allocates sort of automatically:

```
GEN c;  
c = gadd (a, b);
```

- ▶ frees by moving the stack pointer (`avma`, `gerepile`)

- GMP, MPFR, MPC: `mpz_t`, `mpq_t`, `mpfr_t`, `mpc_t`

- ▶ store numbers on the heap
- ▶ require explicit allocation (`mpz_init`, `mpc_init2` → `malloc`)

```
mpz_t c;  
mpz_init (c);  
mpz_add (c, a, b);
```

- ▶ require explicit freeing (`mpz_clear`, `mpc_clear` → `free`)
`mpz_clear (c);`

- PARI

- ▶ has a global precision for the creation of variables
- ▶ each variable implicitly has a given precision
- ▶ works on a best-effort basis for rounding

- MPFR, MPC

- ▶ assign a separate precision to each variable
`mpc_init2 (c, 200);`
- ▶ accept a rounding mode per operation and guarantee the result
`mpc_mul (c, a, b, MPC_RNDND);`

- Both store numbers as arrays of unsigned long int.
- `t_INT` and `mpz_t` have the same endianness.
- `t_REAL` has the other endianness.

Conversion functions provided by Karim Belabas

- From PARI to MP*

- ▶ `void mpz_set_GEN (mpz_ptr z, GEN x);`
- ▶ `void mpq_set_GEN (mpq_ptr q, GEN x);`
- ▶ `int mpfr_set_GEN (mpfr_ptr f, GEN x, mpfr_rnd_t rnd);`
- ▶ `int mpc_set_GEN (mpc_ptr c, GEN x, mpc_rnd_t rnd);`

`x` of type `t_INT`, `t_FRAC`, `t_REAL`, `t_COMPLEX`, as suitable

Semantics: consider `x` as exact, round and return inexact value

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- From MP* to PARI

- ▶ `GEN mpz_get_GEN (mpz_srcptr z);`
- ▶ `GEN mpq_get_GEN (mpq_srcptr q);`
- ▶ `GEN mpfr_get_GEN (mpfr_srcptr f);`
- ▶ `GEN mpc_get_GEN (mpc_srcptr c);`

Semantics: Create `t_REAL` or `t_COMPLEX` with the minimal precision to store `f` or `c` without loss

- Allocate mpfr and mpc numbers on the PARI heap; do not free!
 - ▶ `void pari_mpfr_init2 (mpfr_ptr f, mpfr_prec_t prec);`
 - ▶ `void pari_mpc_init2 (mpc_ptr c, mpfr_prec_t prec);`
 - ▶ `void pari_mpc_init3 (mpc_ptr c, mpfr_prec_t prec_re, mpfr_prec_t prec_im);`
- Emulate PARI precision handling
 - ▶ `void pari_mpfr_init_set_GEN (mpfr_ptr f, GEN x, mpfr_prec_t default_prec);`
 - ▶ `void pari_mpc_init_set_GEN (mpc_ptr c, GEN x, mpfr_prec_t default_prec);`

Allocate on the PARI heap.

For `t_REAL` components, use their own precision.

For `t_INT` and `t_FRAC` components, use `default_prec`.

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Functions available in the PARI library

```
#include "paritwine.h"
```

- MPC

- ▶ GEN pari_mpc_mul (GEN x, GEN y, long prec);
- ▶ all others!

- MPFR

- ▶ GEN pari_mpr_zeta (GEN x, long prec);
- ▶ all others!

- ARB

- ▶ GEN pari_acb_zeta (GEN s, long prec);
- ▶ some more, ask Fredrik!

- CMH

- ▶ GEN pari_cmh_I2I4I6I10 (GEN tau, long prec);
- ▶ GEN pari_cmh_4theta (GEN tau, long prec);
- ▶ GEN pari_cmh_10theta2 (GEN tau, long prec);



```
GEN pari_mpfr_zeta (GEN x, long prec)
{

    mpfr_prec_t p = prec;
    mpfr_t z, z1;

    pari_mpfr_init2 (z, p);
    pari_mpfr_init_set_GEN (z1, x, p);

    mpfr_zeta (z, z1, MPFR_RNDN);

    return      mpfr_get_GEN (z)  ;
}
```

pari_mpfr_zeta

```
GEN pari_mpfr_zeta (GEN x, long prec)
{
    pari_sp ltop = avma;

    mpfr_prec_t p = prec;
    mpfr_t z, z1;

    pari_mpfr_init2 (z, p);
    pari_mpfr_init_set_GEN (z1, x, p);

    mpfr_zeta (z, z1, MPFR_RNDN);

    return gerepileuptoleaf (ltop, mpfr_get_GEN (z) );
}
```

pari_mpfr_zeta

```
GEN pari_mpfr_zeta (GEN x, long prec)
{
    pari_sp ltop = avma;

    mpfr_prec_t p = prec;
    mpfr_t z, z1;

    pari_mpfr_init2 (z, p);
    pari_mpfr_init_set_GEN (z1, x, p);

    mpfr_zeta (z, z1, MPFR_RNDN);

    return gerepileuptoleaf (ltop, mpfr_get_GEN (z) );
}
```

In reality: macro generated



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Use the Foreign Function Interface of GP.

```
LIBPARITWINE_SO="PREFIX/lib/libparitwine.so";  
install ("pari_mpfr_zeta", "Gb", "mpfr_zeta", LIBPARITWINE_SO);
```

- Takes our new function `pari_mpfr_zeta`
- from the installed `libparitwine.so`,
- with one argument of type `GEN` and the default bit precision;
- and calls it `mpfr_zeta` inside GP.

```
read ("PREFIX/share/paritwine/paritwine.gp");  
mpfr_zeta (3)
```

Mini-demo GP

Plans for the future

- Finish wrapping ARB.
- Wrap CM for use in ECPP.
- Wrap FPLLL to test our LLL implementation (requires vectors and matrices).
- Wrap your favourite library.